# APPLICATION OF NUMERICAL METHODS IN SOLVING PROBLEMS OF INTERRELATED HEAT AND MASS TRANSFER AND TRANSFORMATION OF STRUCTURE IN EASILY DEFORMABLE NATURAL SYSTEMS 

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UDC 519.6

The article considers processes of heat and mass transfer with transformation of structure in easily deformable natural disperse systems. A numerical method is used to solve such problems. With this method as a basis, a computational experiment is carried out making it possible to describe the behavior of these systems in a first approximation.

As shown in [1-3], the processes of heat and mass exchange in easily deformable natural disperse systems are associated with transformation of their structure. For these systems, one cannot neglect the effect of change in structure on the processes of transfer. A study of deformation during heat and mass transfer makes it possible to obtain a fuller description of the process itself, thus allowing one to regulate better the technological, agrophysical, and physical-mechanical properties of the given system. To obtain the fullest description of the processes studied, it is necessary to use the physical chemistry of surface phenomena and the physicochemical mechanics of disperse materials. Thus, in [4] on the basis of the physical chemistry of surface phenomena one investigates mass transfer in porous bodies. Shrinkage of a periodic colloidal structure at a low rate of drying is considered. However, because of the complexity of such processes only computations of an approximate character were performed.

To obtain a fuller description of the problems considered, it is necessary to resort to new methods that will make it possible to broaden the range of the problems solved. One of these methods is numerical experiment. Let us single out several areas of its use: first, it should be used instead of experimental investigations that require appreciable time or material input; second, it allows one to play out idealized versions of a process that are not at all experimentally feasible; third, it is possible to solve problems that are described by nonlinear systems of differential equations.

Solution of interrelated processes of heat and mass transfer and transformation of structure in disperse systems can be carried out in two ways. In the first case, on the basis of the thermodynamics of irreversible processes and the mechanics of solid media, a system of differential equations of heat and mass transfer and deformation is written. It is augmented with certain rheological equations. Then, using standard numerical methods (a grid method or the method of finite elements) the written system of differential equations is solved. In the second case, on the basis of studying the physical chemistry of surface phenomena, the physicochemical mechanics of disperse materials, and rheology and processes of heat and mass transfer in disperse systems a model of disperse systems consisting of a finite number of elements is constructed by analogy with molecular dynamics. After that, using developed procedures, algorithms, and programs, specific problems are solved on a computer. This approach is investigated in [5].

Let us consider a body of rectangular shape consisting of a solid phase (skeleton) and a substance in the form of liquid or vapor that fills the pores. The $x$ axis is directed horizontally and the $y$ axis vertically. We assume that the body studied obeys in a first approximation the Hooke rheological equation. On the basis of the thermodynamics of irreversible processes and the mechanics of solid media it is possible to construct a system of

Institute of Problems of Utilization of Natural Resources and the Environment, National Academy of Sciences of Belarus, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal Vol. 72, No. 3, pp. 429-436, May-June, 1999. Original article submitted February 17, 1998.
differential equations that describe the processes of heat and mass transfer and deformation. This approach is widely used in thermoelasticity theory. Thus, in [6-8], applying the thermodynamics of irreversible processes, interrelated equations of thermoelasticity are derived. Further development of this approach is given in [9-11], where general equations are given that describe interrelated processes of heat and mass transfer and deformation. It should be taken into account that the phenomena of heat and mass transfer in disperse capillary colloidal systems are diverse, since they depend on a large number of factors and arise due to many causes. All this presupposes a large number of models of transfer. Therefore, following [12,13], the equations of interrelated heat and mass transfer will be written in a general form. We also avail ourselves of the fact that mechanical processes are able to keep pace with heat and mass exchange, since the period of their relaxation is smaller than in processes of transfer. This makes it possible to disregard the inertial term in equations of motion. Using results of [9-11], we will write a system of interrelated heat and mass transfer and deformation in the form

$$
\begin{gathered}
\frac{\partial T_{\mathrm{dim}}}{\partial \tau}=K_{11} \nabla^{2} T_{\mathrm{dim}}+K_{12} \nabla^{2} W_{\mathrm{dim}}-\frac{E}{3(1-2 \nu)} \frac{\alpha T_{\mathrm{c}}}{\rho c} \operatorname{div} \frac{\partial \mathrm{u}_{\mathrm{dim}}}{\partial \tau}, \\
\frac{\partial W_{\mathrm{dim}}}{\partial \tau}=K_{21} \nabla^{2} T_{\mathrm{dim}}+K_{22} \nabla^{2} W_{\mathrm{dim}}-\operatorname{div} \frac{\partial \mathrm{u}_{\mathrm{dim}}}{\partial \tau}, \\
\Delta \mathrm{u}_{\mathrm{dim}}+\frac{1}{(1-2 \nu)} \operatorname{grad} \operatorname{div} \mathrm{u}_{\mathrm{dim}}=\frac{2(1+\nu)}{3(1-2 \nu)}\left(\alpha \nabla T_{\mathrm{dim}}+\beta \nabla W_{\mathrm{dim}}\right) .
\end{gathered}
$$

Let us go over to the dimensionless form of this system, which allows one to decrease the number of independent parameters and makes consideration of the problem easier. A similar approach is used in [14] for solution of interrelated equations of heat and mass transfer without deformation. Then, in dimensionless form we have the following system of differential equations:

$$
\begin{gathered}
\frac{\partial T}{\partial \mathrm{Fo}}=\nabla^{2} T+D_{1} \nabla^{2} W-D_{2} K_{\alpha} \operatorname{div} \frac{\partial \mathrm{u}}{\partial \mathrm{Fo}}, \\
\frac{\partial W}{\partial \mathrm{Fo}}=D_{3} \nabla^{2} T+\mathrm{Lu} \nabla^{2} W-\operatorname{div} \frac{\partial \mathrm{u}}{\partial \mathrm{Fo}}, \\
\frac{1}{2(1+v)(1-2 v)} \nabla \nabla \mathbf{u}+\frac{1}{2(1+v)} \Delta \mathbf{u}=\frac{1}{(1-2 v)}\left(K_{\alpha} \nabla T+K_{\beta} \nabla W\right),
\end{gathered}
$$

where $D_{1}=\left(K_{12} W_{\mathrm{c}}\right) /\left(K_{11} T_{\mathrm{c}}\right), D_{2}=E /\left(3(1-2 v) \rho c T_{\mathrm{c}}\right), D_{3}=\left(K_{21} T_{\mathrm{c}}\right) /\left(K_{11} W_{\mathrm{c}}\right), \mathrm{Lu}=K_{22} / K_{11}, K_{\alpha}=\alpha T_{\mathrm{c}}, K_{\beta}=$ $\beta W_{\mathrm{c}}$ are dimensionless combinations.

For the heat and mass transfer we avail ourselves of boundary conditions of the third kind on the upper boundary:

$$
\nabla T=-\mathrm{Bi}_{T}\left(T_{\mathrm{s}}-T_{\mathrm{ex.m}}\right), \quad \nabla W=-\mathrm{Bi}_{W}\left(W_{\mathrm{s}}-W_{\mathrm{ex.m}}\right) .
$$

On the remaining boundaries we take $q_{T}=q_{W}=0$.
For the mechanical motion the boundary conditions are written in displacements:

$$
\begin{gathered}
{\left[\frac{v}{(1+v)(1-2 v)}\left(\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}\right)+\frac{1}{1+v} \frac{\partial u_{x}}{\partial x}\right] n_{x}+\frac{1}{2(1+v)}\left(\frac{\partial u_{x}}{\partial y}+\frac{\partial u_{y}}{\partial x}\right) n_{y}=} \\
=\frac{1}{1-2 v}\left(K_{\alpha}\left(T-T_{0}\right)+K_{\beta}\left(W-W_{0}\right)\right) n_{x}, \\
\frac{1}{2(1+v)}\left(\frac{\partial u_{x}}{\partial y}+\frac{\partial u_{y}}{\partial x}\right) n_{x}+\left[\frac{v}{(1+v)(1-2 v)}\left(\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}\right)+\frac{1}{1+v} \frac{\partial u_{y}}{\partial y}\right] n_{y}=
\end{gathered}
$$

$$
=\frac{1}{1-2 v}\left(K_{\alpha}\left(T-T_{0}\right)+K_{\beta}\left(W-W_{0}\right)\right) n_{y} .
$$

We solve this system of differential equations numerically. For its approximation, we avail ourselves of the method of finite differences. Let us take a square grid. For the equations of heat and mass transfer we use a five-point pattern and apply the scheme of central differences. Then we have

$$
\begin{align*}
& \Delta T_{i, j}^{k}=\frac{\Delta \mathrm{Fo}}{h^{2}}\left(T_{i+1, j}^{k-1}+\right.\left.T_{i-1, j}^{k-1}+T_{i, j+1}^{k-1}+T_{i, j-1}^{k-1}-4 T_{i, j}^{k-1}\right)+D_{1} \frac{\Delta \mathrm{Fo}}{h^{2}}\left(W_{i+1, j}^{k-1}+W_{i-1, j}^{k-1}+W_{i, j+1}^{k-1}+\right. \\
&\left.+W_{i, j-1}^{k-1}-4 W_{i, j}^{k-1}\right)-D_{2} K_{\alpha} \frac{1}{2 h}\left(u_{x i+1, j}^{k-1}-u_{x i+1, j}^{k-2}-u_{x i-1, j}^{k-1}+\right. \\
&\left.+u_{x i-1, j}^{k-2}+u_{y i, j+1}^{k-1}-u_{y i, j+1}^{k-2}-u_{y i, j-1}^{k-1}+u_{y i, j-1}^{k-2}\right)  \tag{1}\\
& \Delta W_{i, j}^{k}= \mathrm{Lu} \frac{\Delta \mathrm{Fo}}{h^{2}}\left(W_{i+1, j}^{k-1}+W_{i-1, j}^{k-1}+W_{i, j+1}^{k-1}+W_{i, j-1}^{k-1}-4 W_{i, j}^{k-1}\right)+ \\
&+D_{3} \frac{\Delta \mathrm{Fo}}{h^{2}}\left(T_{i+1, j}^{k-1}+T_{i-1, j}^{k-1}+T_{i, j+1}^{k-1}+T_{i, j-1}^{k-1}-4 T_{i, j}^{k-1}\right)- \\
&-\frac{1}{2 h}\left(u_{x i+1, j}^{k-1}-u_{x i+1, j}^{k-2}-u_{x i-1, j}^{k-1}+u_{x i-1, j}^{k-2}+u_{y i, j+1}^{k-1}-u_{y i, j+1}^{k-2}-u_{y i, j-1}^{k-1}+u_{y i, j-1}^{k-2}\right) . \tag{2}
\end{align*}
$$

For the upper boundary, through which heat and mass exchange with the external medium occurs, we write

$$
\begin{equation*}
T_{i, j}^{k}=\frac{T_{i, j-1}^{k}+\mathrm{Bi}_{T} h T_{\text {ex.m }}}{\mathrm{Bi}_{T} h+1} ; \quad W_{i, j}^{k}=\frac{W_{i, j-1}^{k}+\mathrm{Bi}_{W^{W}} h W_{\text {ex.m }}}{\mathrm{Bi}_{W^{\prime}} h+1} \tag{3}
\end{equation*}
$$

On the remaining boundaries we have

$$
\begin{equation*}
T_{i, j}^{k}=T_{\mathrm{in}}, \quad W_{i, j}^{k}=W_{\mathrm{in}} . \tag{4}
\end{equation*}
$$

The equations of mechanical motion were approximated on a nine-point pattern. Then we represent the difference scheme in the form

$$
\begin{gather*}
u_{x i, j}^{n}=a_{6}\left(a_{1}\left(u_{x i+1, j}^{n-1}+u_{x i-1, j}^{n-1}+u_{x i, j+1}^{n-1}+u_{x i, j-1}^{n-1}\right)+\right. \\
+a_{3}\left(0.25\left(u_{y i+1, j+1}^{n-1}-u_{y i+1, j-1}^{n-1}+u_{y i-1, j-1}^{n-1}-u_{y i-1, j+1}^{n-1}\right)+u_{x i+1, j}^{n-1}+u_{x i-1, j}^{n-1}\right)- \\
\left.-a_{2} \frac{h}{2}\left(K_{\alpha}\left(T_{i+1, j}^{k}-T_{i-1, j}^{k}\right)+K_{\beta}\left(W_{i+1, j}^{k}-W_{i-1, j}^{k}\right)\right)\right),  \tag{5}\\
u_{y i, j}^{n}=a_{6}\left(a_{1}\left(u_{y i+1, j}^{n-1}+u_{y i-1, j}^{n-1}+u_{y i, j+1}^{n-1}+u_{y i, j-1}^{n-1}\right)+\right. \\
+a_{3}\left(0.25\left(u_{x i+1, j+1}^{n-1}+u_{x i+1, j-1}^{n-1}+u_{x i-1, j-1}^{n-1}-u_{x i-1, j+1}^{n-1}\right)+\right. \\
\left.\left.+u_{y i, j+1}^{n-1}+u_{y i, j-1}^{n-1}\right)-a_{2} \frac{h}{2}\left(K_{\alpha}\left(T_{i, j+1}^{k}-T_{i, j-1}^{k}\right)+K_{\beta}\left(W_{i, j+1}^{k}-W_{i, j-1}^{k}\right)\right)\right), \tag{6}
\end{gather*}
$$

where $a_{6}=1 /\left(4 a_{1}+2 a_{3}\right), a_{1}=1 /(2(1+v)) ; a_{2}=1 /(1-2 \nu) ; a_{3}=a_{1} a_{2}$.

Fig. 1. Typical change in the shape of a body during heat and mass transfer.
On the boundary the difference expressions result from the following general difference relations:

$$
\begin{gather*}
\left(a_{4}\left(u_{x i+1, j}-u_{x i-1, j}+u_{y i, j+1}-u_{y i, j-1}\right)+a_{5}\left(u_{x i+1, j}-u_{x i-1, j}\right)\right) n_{x}+ \\
+a_{1}\left(u_{x i, j+1}-u_{x i, j-1}+u_{y i+1, j}+u_{y i-1, j}\right) n_{y}=2 h a_{2}\left(K_{\alpha}\left(T_{i j}-T_{0}\right)+K_{\beta}\left(W_{i j}-W_{0}\right)\right) n_{x},  \tag{7}\\
a_{1}\left(u_{x i, j+1}-u_{x i, j-1}+u_{y i+1, j}-u_{y i-1, j}\right) n_{x}+\left(a_{4}\left(u_{x i+1, j}-u_{x i-1, j}+u_{y i, j+1}-u_{y i, j-1}\right)+\right. \\
\left.+a_{5}\left(u_{y i, j+1}-u_{y i, j-1}\right)\right) n_{y}=2 h a_{2}\left(K_{\alpha}\left(T_{i j}-T_{0}\right)+K_{\beta}\left(W_{i j}-W_{0}\right)\right) n_{y}, \tag{8}
\end{gather*}
$$

where $a_{4}=2 v a_{3} ; a_{5}=2 a_{1}$.
Moreover, immobility of the center of the body as a result of symmetry is assumed, which allows one to introduce an additional condition. We assume that at the center of the body $u_{x i, j}=0$.

The heat- and mass-transfer equations were solved by an explicit method using expressions (1)-(4):

$$
T_{i, j}^{k}=T_{i, j}^{k-1}+\Delta T_{i, j}^{k} ; \quad W_{i, j}^{k}=W_{i, j}^{k-1}+\Delta W_{i, j}^{k} .
$$

The equations of motion in displacements (5)-(8) were solved by the iteration method of under relaxation [15] at each step $\Delta$ Fo. To stop the iterative procedure, we used the expression $\max \left(\frac{\left|u^{n}-u^{n-1}\right|}{\left|u^{n}\right|}\right)<\varepsilon$.

Performing calculations, it is possible to obtain the distribution of the quantities $T, W, \sigma_{x x}, \sigma_{y y}, \sigma_{x y}$, and $\mathbf{u}$ in space and time, which makes it possible to describe the behavior of the body. A typical change in the shape of the body in the process of heat and mass transfer is shown in Fig. 1. Numerous computational experiments performed on this model give a qualitative picture of the deformation in time of a body that experiences heat and mass transfer. In the initial period, as a result of a decrease in the moisture content, the upper surface contracts. This leads to the appearance of tensile stresses on the upper and side surfaces. The lower surface is also stretched, and this causes a slight increase in its dimensions in comparison with its initial size. The edges of the upper surface are bent. Further heat and mass transfer decreases the moisture content, now over the entire body, and this leads to shrinkage of the entire specimen. However, as long as the gradients of the fields increase, the deformation of the shape also increases. This causes the lower surface to bend upward. After that, the shape ceases to change. When the gradients of the fields decrease, the surfaces begin to straighten out slowly, and the body continues to dry as before. A further decrease in the gradients leads to a decrease in the deformation of the shape, and at the end of the process of heat and mass transfer the body takes its original shape but of a smaller size that corresponds to the new moisture content, temperature, and coefficients of shrinkage and expansion.

Let $W_{\mathrm{f}}$ be the moisture content of the body at which the moisture content of its lower surface is equal to $0.1 \%$ of the initial value. Now, we estimate the time of termination of the process of heat and mass transfer both in the presence of deformation of the body and without it. For the end of the process we take the time when the moisture content becomes smaller than $W_{\mathrm{f}}$. Calculations were carried out for the following values of the characteristics: $D_{1}=0.1 ; D_{2}=1 ; D_{3}=0.1 ; \mathrm{Lu}=1 ; K_{\alpha}=0.005 ; \mathrm{Bi}_{W}=\mathrm{Bi}_{T}=1 ; T_{0}=W_{0}=1 ; T_{\text {ex.m }}=W_{\text {ex.m }}=0$. In the absence of deformation of the body, the process terminates at $\mathrm{Fo}_{\mathrm{f}}=5.5$. In the presence of deformation, the time of termination of the process of heat and mass exchange depends on the cocfficients of shrinkage and expansion

TABLE 1. Dependence of $\mathrm{Fo}_{\mathrm{f}}$ on the Shrinkage Coefficient for Different Values of the Poisson Coefficient

| $K \beta$ | Fof |  |  |
| :---: | :---: | :---: | :---: |
|  | $v=0$ | $v=0.3$ | $v=0.49$ |
| 0.01 | 5.7 | 5.8 | 5.8 |
| 0.05 | 6.3 | 6.5 | 6.6 |
| 0.1 | 7.0 | 7.4 | 7.6 |
| 0.2 | 8.3 | 9.1 | 9.5 |
| 0.3 | 9.6 | 10.7 | 11.4 |
| 0.4 | 10.8 | 12.3 | 13.2 |
| 0.5 | 12.1 | 13.9 | - |



Fig. 2. Curves of the kinetics of moisture content for different values of $K_{\beta}$ : 1) $\left.\left.\left.\left.K_{\beta}=0,2\right) 0.05,3\right) 0.2,4\right) 0.4,5\right) 0.6$.
and the Poisson coefficient. The dependence of $\mathrm{Fo}_{\mathrm{f}}$, which characterizes the end of the process of heat and mass transfer, on the coefficient of shrinkage for different values of the Poisson coefficient is displayed in Table 1.

As the moisture content of the body. we take its average value $W_{\mathrm{av}}=\frac{1}{N} \sum_{l=1}^{N} W_{l}$. We adopt $v=0.3$, and we leave the other parameters without change. The kinetics of drying for a nondeformed and a deformed body for different coefficients of shrinkage is presented in Fig. 2.

We consider now the dynamics of the temperature and moisture-content fields. Here $K_{\beta}=4$ and the remaining parameters do not change. Curves of the distribution of the moisture content and temperature at the center of the specimen for different values of Fo for both a deformed and a nondeformed body are presented in Fig. 3. Comparison of the curves indicates a difference in the distribution of the heat and moisture fields. Computations showed that they decrease with decrease in the coefficients of shrinkage and expansion.

The difference in the behavior of the moisture-content fields of a deformed and a nondeformed body is associated with the mobility of the skeleton. In a deformed body, not only moisture but also the solid phase of the substance passes through the control volume, and this decreases the fraction of liquid and vapor in the flow. From Table 1 it is seen that the time of the end of the process also depends on the Poissson,coefficient. It is known that at $v=0$ deformation of the body is associated with a change in its volume, and at $v=0.5$ a change in the shape occurs without a change in the volume of the body. Thus, prescribing different values of the Poisson coefficient, we thereby predetermine various behavior of a porous body in deformation. Therefore, a difference in the transferof mass is associated with a change in the relative content of pores in the body.

Consider the stresses that arise in the body. It should be noted that it is very difficult to obtain experimentally graphs of the distribution of the stress-tensor components over the coordinates and their dynamics in time. Therefore, finding even qualitative distributions is of some interest. We took the dimensionless Young's modulus to be equal to one. Typical distributions of the stresses $\sigma_{x x}, \sigma_{y y}$, and $\sigma_{x y}$ over the $x$ coordinate for different





$$
\begin{array}{llllll}
-1 & -2 & --3 & -4 & +-5 & x-6
\end{array}
$$

$$
\begin{array}{cccccc}
*-7 & 0-8 & \Delta-9 & 0-10 & 0-11 & --12
\end{array}
$$

Fig. 3. Distribution of temperature and moisture content at different values of Fo in the presence ( $\mathrm{a}, \mathrm{c}$ ) and absence ( $\mathrm{b}, \mathrm{d}$ ) of deformation: 1) $\mathrm{Fo}=0.01$, 2) $0.1,3) 0.3,4) 0.5,5) 0.7,6) 0.9,7) 1.2,8) 1.5,9) 2,10) 3,11) 5,12) 9$.
layers along the $y$ coordinate at $\mathrm{Fo}=0.3$ are presented in Fig. 4. The other parameters remain as they were. The counting of the layers begins from the lower surface ( 0 is the lower surface, 19 is the upper surface). As one would expect from the conditions of the problem, the components of the tensor of stresses are symmetric about the center of the body. The component $\sigma_{x x}$ acquires its maximum value on the upper and lower surfaces, $\sigma_{y y}$ is greatest in absolute value on the side surfaces, and the shear stress $\sigma_{x y}$ attains its maximum absolute value within the body between the center and a side surface. In the central portion of the body and on all surfaces $\sigma_{x y}=0$.

The symmetric form of the stresses allows one to consider the distribution of the components along the coordinate $y$ for different values of Fo at the center of the body in the case of $\sigma_{x x}$ and $\sigma_{y y}$ and within the body between the center and a side surface in the case of $\sigma_{x y}$. Thus, in the case of $\sigma_{x x}$ in the initial period of heat and mass exchange stresses arise in the upper layers and decrease sharply toward the bottom of the body. In this period they increase rapidly on the upper surface. After a certain time, as the stresses increase in the upper layers, the stresses in the lower layers also begin to increase. On the lower boundary this occurs most vigorously. At the center the stresses acquire a negative, though small, value. As a result we obtain that the stresses fall from the upper surface into the interior and their smallest value is at the center, but then they increase again from the center to the lower surface. However, on the upper surface they are still higher than on the lower onc. Subsequently, they fall on the upper surface and increase on the lower one. At the center the stresses remain minimum as before. As Fo increases, the stresses on the upper surface become smaller than on the lower one. After this, the stresses on the lower surface, which up to now increased, begin to decrease, just as in the upper layers. Then they decrease in absolute value on all planes, tending to zero at the end of the process.


Fig. 4. Distribution of stresses over the $x$ coordinate in different layers along the $y$ axis: a) $\sigma_{x x}$; b) $\sigma_{y y}$; c) $\sigma_{x y}$. The figures at the points denote the numbers of the layers.

The stresses $\sigma_{y y}$ at the center of the body take their smallest absolute values on the lower surface and increase monotonically toward the upper surface, but on the surface itself they again fall sharply to zero. The negative sign of $\sigma_{y y}$ indicates that the body shrinks at its center. But closer to the side surfaces the sign of $\sigma_{y y}$ is changed. In the lower regions the stresses are equal to zero, but they increase closer to the center and take on maximum positive values at the center. From the center upward the values decrease and become negative. But on the surface itself they are equal to zero. This indicates that the lower edges of the body move upward and the upper downward. At the center of the body maximum values are attained at the beginning of the process of heat and mass exchange. As Fo increases, the absolute values of $\sigma_{y y}$ tend to zero. On the side surfaces they first increase, attain a maximum at a certain Fo , and then fall to zero by the end of the process of heat and mass exchange.

The quantities $\sigma_{x y}$ at the beginning of heat and mass exchange increase in absolute value in the upper half of the body, while they are equal to zero in the lower half. As Fo increases, the shear stresses in the upper half of the body also increase. At a certain Fo stresses also appear in the lower half, but their sign is opposite to that of the stresses in the upper half. Subsequently, $\sigma_{x y}$ in both the upper and lower parts of the body continue to increase in absolute value. At a certain Fo, in the upper half of the body $\sigma_{x y}$ attain their maximum value, after which they begin to decrease. In the lower half they increase as before. Then, Fo at which in the lower portion of the body $\sigma_{x y}$ attain a maximum in absolute value is reached, after which they fall, just as in the upper half, tending to zero at the end of the process.

Calculations also showed that the stresses in the body increase with increase in its linear dimensions. It should be noted that upon displacement of the particles of the skeleton, in addition to the forces that act on the solid phase, a pressure $P$ appears in the liquid phase of the body. The gradient of the latter will cause filtrational motion of the moisture. Therefore, for a more complete description it would be necessary to introduce and consider, along with the fields of $T$ and $W$, the distribution of the pressure $P$ in the body.

Although the given model does not allow one to calculate the process of cracking, knowing the strength of the body it is possible to determine the conditions under which it occurs.

For a more complete and accurate description of the processes of heat and mass transfer with transformation of the structure in easily deformable natural disperse systems within the framework of the mechanics of solid media, in addition to the introduction of the additional variable $P$ it is necessary to take the coefficients $K_{11}, K_{12}, K_{21}$, $K_{22}, c, \rho$ not as constant quantities, but as functions that depend on $T$ and $W$ and the changing structure of the body. Moreover, it is necessary to use a more complex rheology of the body that would describe its mechanical behavior more accurately.

However, the present model in a first approximation describes the behavior of a deformable body in the course of heat and mass transfer and allows one to obtain the distribution of the temperature and moisture-content fields and the fields of deformations and stresses, to make a comparative analysis of the processes of heat and mass transfer with deformation and without it, to determine the time of termination of the processes of heat and mass transfer, to follow the change in the shape of the body in time, and to evaluate possible conditions of cracking.

## NOTATION

$T_{\text {dim }}$, temperature, K ; $W_{\text {dim }}$, moisture content, $\mathrm{kg} / \mathrm{kg} ; \mathrm{u}_{\text {dim }}$, displacement, $\mathrm{m} ; T$, dimensionless temperature; $W$, dimensionless moisture content; $u$, dimensionless displacement; $K_{11}, K_{12}, K_{21}, K_{22}$, coefficients of heat and moisture transfer, $\mathrm{m}^{2} / \mathrm{sec},\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right) / \mathrm{sec}, \mathrm{m}^{2} /(\mathrm{sec} \cdot \mathrm{K}), \mathrm{m}^{2} / \mathrm{sec} ; E$, Young's modulus, Pa; $v$, Poisson modulus; $c$, specific heat, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K}) ; \alpha$, temperature coefficient of expansion, $1 / \mathrm{K} ; \beta$, shrinkage coefficient; $\rho$, density, $\mathrm{kg} / \mathrm{m}^{3} ; D_{1}, D_{2}, D_{3}, \mathrm{Lu}, K_{\alpha}, K_{\beta}$, dimensionless combinations; $T_{\mathrm{c}}$, characteristic temperature, $\mathrm{K} ; W_{\mathrm{c}}$, characteristic moisture content, $\mathrm{kg} / \mathrm{kg} ; \mathrm{Bi}_{T}$, temperature Biot number; $\mathrm{Bi}_{W}$, mass-exchange Biot number; Fo , Fourier number; $T_{\mathrm{s}}, W_{\mathrm{s}}$, dimensionless values of the temperature and moisture content on the surface; $T_{\mathrm{in}}$, $W_{\mathrm{in}}$, dimensionless temperature and moisture content of internal points adjoining boundary points; $T_{0}, W_{0}$, dimensionless initial temperature and moisture content of the body; $T_{\text {ex.m }}, W_{\text {ex.m }}$, dimensionless temperature and moisture content of the external medium; $q_{T}$, heat flux, $\mathrm{W} / \mathrm{m}^{2} ; q_{W}$, flux of moisture, $\mathrm{kg} /\left(\mathrm{m}^{2} \cdot \mathrm{sec}\right) ; \sigma_{x x}, \sigma_{y y}, \sigma_{x y}$, dimensionless components of the tensor of stresses; $h$, step of the grid; $k$, number of the layer in time; $n$, number of the iteration in the iteration method; $W_{\mathrm{av}}$, dimensionless average moisture content of the body; $W_{l}$, dimensionless moisture content at the node $l ; N$, number of nodes; $\varepsilon$, a certain small constant number; $\tau$, time, sec; $u_{x}, u_{y}$, dimensionless components of the displacement; $n_{x}, n_{y}$, direction cosines; $i$, number of the node along the $x$ axis; $j$, number of the node along the $y$ axis; $l$, global number of the node. Subscripts: dim, dimensional; s, surface; ex.m, external medium; f, final; av, average value; $c$, characteristic; in, internal; 0 , initial.

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